Lesson Research Proposal

Date of lesson: 13/02/19
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1. Title of the Lesson: The Power that Lies Beneath

2. Brief description of the lesson

This lesson demonstrates the use of assessment for learning techniques to identify students' understanding of a concept to inform progress in the classroom. The lesson introduces students to the index laws $a^{-p} = \frac{1}{a^p}$ and $a^0 = 1$ through investigating a problem and recognising a geometric pattern.

3. Research Theme

At Woodbrook College, the third strand of our School Self-Evaluation cycle (SSE) is Assessment for Learning (AfL). The school has set specific targets; one of these targets being to increase the number of students who regularly edit their own work. This was picked in an effort to enhance student's motivation to learn and also to increase the individual student's self-expectations of achievement. Another target was to increase task focused feedback, in an effort to reduce the number of students confused by teachers. A strategy employed to assist in achieving this target was to promote the levels of reflection by the student on their own work and to encourage a sense of ownership and responsibility of their own work. With this in mind, the Mathematics Department in Woodbrook College have considered how to incorporate assessment for learning techniques into the classroom which will support peer reflection and editing of student's own work. The Mathematics Department regularly promotes group work, self-reflection, peer assessment and verbalising of their Mathematics.

From a teaching point of view, in Woodbrook College, real efforts are being made to increase collaboration and common assessment and this is reflected in our School self-evaluation targets. Participation in lesson study affords teachers in the Mathematics Department an opportunity to collaborate through structured dialogue, build on the practice of knowledge sharing and engage with professional development to gain a deeper understanding of assessment for learning in the teaching of Mathematics.

4. Background & Rationale

Topic - Indices
Year - 2nd Year
Level - Mixed ability

There is a general consensus in our Mathematics Department that we have a limited approach to introducing the concept of Indices and we hope to explore a wider scope of delivery for this topic. Through examining students work and observations in the classroom students frequently mishandle exponents, in particular negative exponents all the way through to Leaving Certificate. We have chosen to focus on an alternative approach in second year to make sense

of exponents for students and introduce the topic in a more meaningful way.
On a national level, the Chief Examiners Report in 2015 on Junior Certificate Mathematics reference general carelessness in relation to Exponents "*Students should be particularly careful with signs, powers, and the order of operations*".

Assessment for Learning (AfL) – As a Department, we feel there is a lack of good resources and examples of good practice for using assessment for learning in Mathematics. As AfL forms the basis for our schools' current strand of our SSE, we wanted to consider meaningful approaches and strategies to support deeper learning for students.

For this lesson the Mathematics Department have chosen the use of multiple choice questions and show me board answers as a means for inform the teacher of the general level of learning in the classroom. Questions will be designed to highlight common misconceptions, offer different forms of correct solutions and leave room for 'none of the above' responses. The teacher can use this snapshot of the classroom to inform the progress of the lesson, providing extensions for students who are ready and scaffolds for students who are struggling.

5. Relationship of the Unit to the Syllabus

Related prior learning Outcomes	Learning outcomes for this unit	Related later outcomes
Students should be able to: use and apply the rules for indices (where a ∈ Z, a ≠ 0; p, q ∈ N): • $a^p . a^q = a^{p+q}$ • $\frac{a^p}{a^q} = a^{p-q}$ • $a^0 = 1$ appreciate the order of operations including the use of brackets use the equivalence of fractions, decimals and percentages to compare proportions use tables, diagrams and graphs as tools for representing and analysing linear, quadratic and exponential patterns and relations	Students should be able to: use and apply the rules for indices as outlined in the JC Syllabus (Topic 3.2) operate on the set of irrational numbers R \ Q use the notation a½, a ∈ N express rational numbers ≥1 in the approximate form $a\ x\ 10^n$,where a is in decimal form correct to a specified number of places and where $n = 0$ or $n \in$ N express non-zero positive rational numbers in the approximate form form $a\ x\ 10^n$, where $n \in$ Z and $1 \leq a < 10$ compute reciprocals	Students should be able to: (Number): Solve problems that involve compound interest Explore patterns and formulate conjectures Explain findings Justify conclusions Communicate mathematics verbally and in written form (Algebra): Show that relations have features that can be represented in a variety of ways Use the representations to reason about the situation from which the relationship is derived and communicate their thinking to others

6. Goals of the Unit

Students will understand what the effect of an exponent on a number means, to recognise the connection between multiplication and addition of values, to be aware of how to graphically represent values that are in index form and to understand what connection lies between negative exponents and reciprocal values. Students should make the connection between an exponential pattern and the index laws. The relationship between a number, its reciprocal and the index law should be clear to students.

7. Unit Plan

Lesson	Brief overview of lessons in unit
1-3	Use simple problems to develop and formulate (Rules 1- 3)
4	Research Lesson
5-6	Introduction of Rational Exponent (Rules 6 – 7)
7	Introduction of remaining two index rules (Rules 8-9)
8-9	Solving of equations with indices
12-13	Scientific notation.

8. Goals of the Research Lesson:

Mathematical goals:

1. Students will realise that a^{-p} has its own applications and value, not only its properties illustrated in Rule 2.
2. Students will deduce that $a^0 = 1$.
3. Students will recognise a connection to exponential patterns.
4. Students will be able to create the reciprocal pattern and compare exponents.

Key Skills and Statements of Learning

1. Students will be able to verbalise their own learning.
2. Students will be able to see patterns and relationships.

Assessment for Learning:

1. Teacher will use multiple choice diagnostic testing
2. Students will engage in think, pair, share, discussion to assist in self-editing
3. Teacher will ask effective questions

9. Flow of the Research Lesson:

Steps, Learning Activities, Teacher's Questions and Expected Reactions	Teacher Support	Assessment
Introduction 10 minutes starter activity	Present two multiple choice questions within the laws of indices (multiplication & division). The solutions include answers arising from common misconceptions. Structured class discussion if students choose wrong answers to allow them to correctly identify the solution. Summary of the learning	(a)Show me boards give a snapshot of answers highlighting: misconceptions, (stage of students' progress) (b) Observers are recording the variety of approaches - who uses the laws, who manually calculates the answer
Posing the Task & Individual Work (20 mins) Present the problem 2 mins 5 mins 1 min 5 mins	 Problem will be delivered through a Power-point and handout Students will have 5 minutes individual thinking time to attempt the problem Teacher asks for show me board response to question (i) Based on responses some supporting questions to particular students may be needed (differentiated) (Groups may be re-structured if needed)	 (a) Teacher circulates, notes how students are approaching the question. (b) Observers note the different approaches taken and the common misconceptions. Take photographs where appropriate. (a)Show me boards give a snapshot of answers highlights: misconceptions, (stage of students' progress) (a)Circulate, monitor discussions to see what stages students are at.

1 min 5 mins 1 min	Students have 5 minutes' pair share time to discuss their approaches Teacher asks for show me board response to question (i) Student has 5 minutes to work on the problem (part ii) Teacher asks for show me board response to question (ii)	(b)record key quotes from discussion & photos of the work (a) Responses allow teacher to know if it is appropriate to move on. Teacher can identify struggling students. (a) Teacher circulates notes how quickly students are approaching the question (b) Observers note the different approaches taken by students and the common misconceptions (a)Show me boards give a snapshot of answers highlights: misconceptions, (stage of students' progress) Responses inform teacher of students thinking for discussion
Ceardaíocht /Comparing and Discussing 20 mins	Teacher displays table on board. Students asked to come up at various stages to write in an answer. Positioning of answers is key to ensuring the pattern appears in a logical fashion. Discussion around each answer, its equivalent forms and how the students got the answer.	(a)Use of effective questioning to lead students to recognizing the 2 concurrent patterns (b) Record responses among the class to the work presented. Look for signs of 'aha' moment. (a)Encouraging student to clarify/explain the method to getting their answer. (b) Record interesting comments

Summing up & Reflection 10 mins	Students are invited to consider and record their key learning. Summary of key points on the board. Students record additional points. Students fill out the reflection sheet.	(a) Circulate and observe what they are writing. (b) note the key comments – do the students list the desired key points initially.

10. Board Plan

2003	…	2016	2017	2018	2019	2020	2021	2022	…	2033
$\frac{8}{14348907}$ 8.57×3^{-15} Million tonnes	.....	0.296 $\frac{8}{27}$	0.889 $\frac{8}{9}$ Million tonnes	2.667 $\frac{8}{3}$ Million tonnes	8 Million tonnes	24 Million tonnes	72 Million tonnes	216 Million tonnes	…	114791256 Million tonnes
$8x\frac{1}{3}x\frac{1}{3}\ x\ ....x\frac{1}{3}$ (15 times)	.....	$8x\frac{1}{3}\ x\frac{1}{3}x\frac{1}{3}$	$8x\frac{1}{3}x\frac{1}{3}$	$8x\frac{1}{3}$	8	8 ×3	8×3×3	8×3×3×3		8×3×…×3×3 (15 times)
$8 \times (\frac{1}{3})^{15}$	.....	$8\times (\frac{1}{3})^{3}$	$8x(\frac{1}{3})^{2}$	$8x(\frac{1}{3})^{1}$	$8x(\frac{1}{3})^{0}$					
8×3^{-15}	…	8×3^{-3}	8×3^{-2}	8×3^{-1}	8×3^{0}	8×3^{1}	8×3^{2}	8×3^{3}		8×3^{15}

11. Evaluation

AfL

Did the lesson allow students an opportunity to edit their own work?

Each time the multiple choice diagnostic was issued, students had an opportunity to discuss with their groups the method they had chosen and how they had arrived at the answer. Since no correct answers were given, students had to convince each other of the validity of their thinking. This promoted rich discussions among the students, followed by an opportunity for students to return to their work and make improvements.

Did the results of the multiple choice diagnostics inform the progress of the lesson?

By repeating the diagnostic it was clear when to move the lesson on to the next concept. Students became more comfortable discussing their solutions and making edits as the lesson progressed.
There were less misconceptions arising each time as the diagnostic was repeated which suggested students were learning from the discussion with each other and teasing out where the misconceptions were arising from. The teacher was able to quickly identify the two students who were struggling with the starter activity and tease out through questioning them and their peers where the misconception was. Repeating the diagnostic allowed the teacher to monitor the overall level of understanding at a glance and identify students who needed additional support.

Did the students learn from discussion with their peers and use this to further their learning?

At various stages during the lessons students had an opportunity to discuss their solution and verbalise their thinking. The students were confident discussing with their peers in small groups and quick to engage in debates over the correct approach. When it came to discussing these ideas in the full group students were more reluctant to share these ideas. There were several situations where students who had made mistakes in the early task were convinced by their peers of why their solution was incorrect and why an alternative approach was needed. These mistakes were rarely repeated in the follow up tasks indicating that learning had occurred. It was also interesting to note the discussion around the question where two equivalent correct answers were presented in the diagnostic. A large amount of students concluded after discussing with their group that both A and B were correct and presented both together on their boards.

Did the lesson encourage reflection among students?

Although students seemed to be learning through the discussion it would have been beneficial for them to have spent time recording key learning points during the lesson, for example, misconceptions and new ideas, during the discussions. Students did record the overall summary at the end of the lesson but it may have been beneficial to record more information during the discussions in order to tailor the learning experience to the needs of the individual.

Evaluation of Lesson Goals
Do students understand that a^{-p} has its own applications and value?
Did students identify an exponential pattern?
Did students deduce that $a^0 = 1$
Were students able to create the reciprocal pattern?
Were students able to compare exponents?

During the group discussion students quickly identified an exponential pattern as they were recording solutions on the board. They also recognised that the exponents moved in a linear pattern without prompting. They naturally included the 3^0 term in the pattern.

Surprisingly they initially struggled with the concept that ÷ 3 was equivalent to × $\frac{1}{3}$ but once that was explored further they were able to create the reciprocal pattern easily. Student comments were "*I knew how to do it, it's just 15 years ahead so multiply by three 15 times." "triple eight million, you have to triple it 15 times"* "*You just go back now so divide instead of multiply".*

They made the link between the proportional patterns but not all students got quite as far as generalising the rule.

Part 1:
Some misconceptions for the first part were:

- For two years 8 x 3 x 2 again when questioned the student corrected
- 3 x 15 + 8
- 15 x 8

All of these were corrected during the small group discussion among peers.

Most common approaches taken:

- Tally to record the number of times three was multiplied
- List of each total for each year

Part 2:
An interesting misconception for the second part was:

- $8\,x3^{-15}$ showing that use of the inverse operation was recognised but applied incorrectly.

Most common approaches

- $\frac{8}{3^{-15}}$ showing deep understanding of the solution to part 1
- 8÷3÷3÷3…

12. Reflection – The Lesson

What had we hoped to observe during the lesson?

We hoped to see the use of multiple choice diagnostics informing the pace at which the lesson progressed. We hoped the students would, through discussion, self-edit their work and draw conclusions based on these discussions.

We were hoping students would enjoy the lesson, engage with group work in a productive manner and reflect on their learning.

What was actually observed during the lesson, by the team members and others?

The starter activities highlighted that some students did have misconceptions about the previous index laws. Through group discussion these students were afforded an opportunity to work out the misconceptions before engaging with the new material.

Students took a variety of approaches to solving the lesson, several misconceptions were identified in the first round of the 'show me' display. During the first discussion, most students corrected their initial misconceptions.

A highlight was that after the first problem had been discussed some students who had manually listed each step in part (i), recognised the pattern of 8 x 3 x 3 x3 x 3… (15 times) then went straight to $8 \div 3^{15}$ for part (ii) without needing to list the steps.

There were several rich discussions from the students, in particular, weaker students seemed to gain a lot from the discussion. There were cases however where dominant students with incorrect solutions convinced correct students that their correct answer was wrong. Perhaps more thought on how to address this for next time is needed. The higher ability students challenged by having to verbalise their learning in a way that their peers could understand. In addition, the extension tasks provided additional challenges for some students who displayed good understanding of the solution and had explored the main task in detail.

In conclusion, deep exploration of the problem, without giving the answer, produced rich discussion across all ability levels and a focus on exploring multiple approaches led to deeper understanding of the process rather than jumping to a quick solution.

What were the major points raised during the post-lesson discussion, and the team's own opinions?

It was agreed that the main successes of the lesson were that every student gets to participate and share their learning and that the diagnostic testing encouraged every student to give a response each time. No student was able to avoid engaging as a response was required from all students.

The observers noted new learning for them was that students struggle with individual problem time. While this is known in theory observing the interaction of students provided a deeper understanding of this point. They concluded that it is important they are encouraged to work alone at times and to be allowed to struggle with problems. Individual thinking time and problem solving also transfers to exam skills and it is worth being mindful of allowing students the freedom to take this time to engage with problems. In addition, wait time is so important, it is difficult to allow students sufficient thinking time without jumping in to assist. As a department an agreement was made that this is worth focusing on for future improvement of teaching and learning.

Reflection – The Lesson Study Process

Lesson study was a great chance to collaborate with colleagues and share different teaching methods and ideas. It is rare that we have an opportunity to reflect on our teaching and learning. It allowed us to work together to gain an understanding of what the key priorities are for us as a department. We also had an opportunity to consider teaching and learning from the perspective of a student, which highlighted misunderstandings that students can have. While we focused our investigation on a particular concept, now that we are familiar with the process we will be able to consider new priorities and new teaching and learning goals in the future.

Engaging in the process has encouraged us to speak to each other more frequently about our teaching and learning, for example simply exchanging ideas for different topics and discussing our current classroom practice. At the end of the process we all agreed that we didn't leave the kids enough thinking time or discussion time in our lessons so we are planning on implementing this in our classroom going forward. We also became aware of how much learning can occur from pupil discussion where previously we would not as individuals have been willing to risk group-work.

It was interesting taking the time to look at a topic we find students struggle to understand. We were able to take time to design a lesson so that it would promote inquiry based learning, where students would use their prior knowledge of the topic to solve a problem. While this was a long process for one concept, we feel we gained invaluable experience of how to plan for student-centred learning and going forward will be able to incorporate some of this new learning in other areas. Overall we felt that collaborative, structured planning allows for deeper learning in the classroom and that as teachers we should not fear discussion among students once it has a focus and well planned learning intention.

At the end of the process we reflected as a team on our lesson, observers and the teacher gave their opinions on the lesson from different views of the classroom. Observing students working without the pressures of teaching was a rare opportunity which provided much insight into how students think and engage with a problem. We learnt that when students got to explore their options for answering a question and had time to find their own mistakes they showed remarkable confidence and were quite capable of self-editing.

With regards our own practice we felt that the experience has allowed us a genuine opportunity to reflect on our practice and consider simple but effective ways to change our practice for the benefit of our students. The process was challenging but also very rewarding. Investing so much time on top of a busy schedule is a big commitment, however, the support of our colleagues provided the motivation to continue with the process and we discovered that in order to achieve change to our practice we needed this time to discuss, plan and reflect. We are really glad we went through it and would be interested in signing up again having seen the benefits first hand.

Appendix 1 – Observation Template

Stage of Lesson	Observers should be	Possible student answers to look for.
Introduction Starter Activity Multiple choice	Looking for who is using the law and who is using calculators.	Approaches Question 1: 1) A 2) B 3) C 4) D 5) Other/Guess Question 2: 1)A 2)B 3)C 4)D 5) Other/Guess Comments on student's work:
Goals	Understand the effect of an exponent on a number. Recognise the connection between multiplication and addition of values	
Posing Task + Individual work Pollution problem Part 1.	Looking for method student uses to solve their question to get their answer.	Approaches 1)8 x 15 2)8 x 3 x 15 3)8 x 3^{15} 4)Other Method 5) 3^{15} 6)Pupil who got answer 3 by using 8 x 3 … (15 times)
Discussion time	Record Quotes from discussion of how	Comments on discussion (Think Pair Share/

	students are justifying their work.	Students Reasoning / Misconceptions) Comment on AFL Most common misconceptions
Pollution Problem Part 2. Discussion time for Q2.	Looking for method student uses to solve their question to get their answer. Record Quotes from discussion of how students are justifying their work	Approaches 1) 8 / 3 (15 times) (In fraction format) 2) 8 / 3 (15 times) (In scientific notation) 3) 8mil - 114791256 4)8- (15 x 3) 5)Other Method 6) Pupil who got answer one or two using 8×3^{-15} Comments on discussion (Think Pair Share/ Students Reasoning / Misconceptions) Comment on AFL
Goal to meet	Understand what connection lies	

	between negative exponents and reciprocal values	
Comparing and Discussing Working out table on the board Calculations going forward years Converting to indices Calculations going backwards Converting to indices	Recording responses among the class to the work which is being presented.	Comments made during discussion
Goals to meet	Understand what connection lies between negative exponents and reciprocal values. Students should make the connection between an exponential pattern and the index laws.	

	The relationship between a number, its reciprocal and the index laws should be clear to students.	

Appendix 2 – The Problem

Every year the amount of plastic in the ocean triples!!
In 2019, there is 8 million tons of plastic in our ocean. That is the equivalent to 5 grocery bags filled with plastic for every foot of the coastline in the world!

(i) How many tons of plastic will be there in 15 years?

(ii) How many tons of plastic were there 15 years ago?

In 2019, there is 8 million tons of plastic in our ocean. That is the equivaler
filled with plastic for every foot of the coastline in the world!

(i) How many grocery bags could you fill with plastic in 15 years?

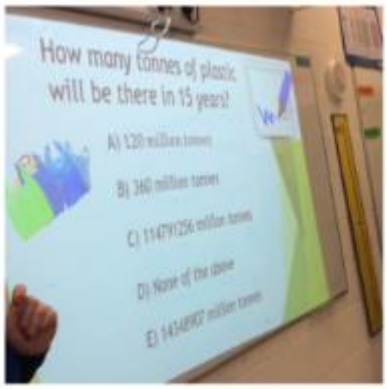
(ii) 1 ton = 1000kg. How many kg of plastic was there in the ocean 5 years ago? Give your answer correct to 2 decimal places.

(iii) 1 ton = 2240 lbs. (pounds). How many lbs. of plastic will there in 15 years? Give your answer correct to 1 decimal place.

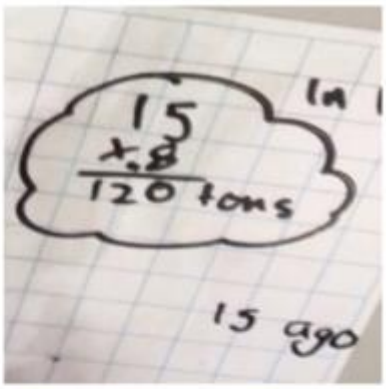
(iv) 1 foot = 30.48cm and 1 mile = 5280 feet. There are 598675.97km of coastline in the world. How many miles of coastline is there? Give your answer to the nearest mile.

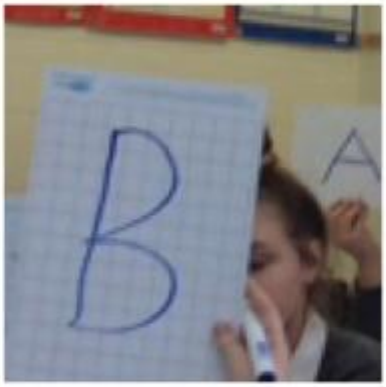
(v)

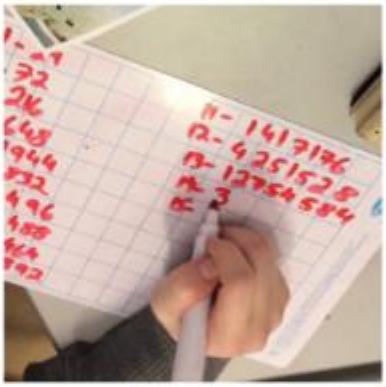
Appendix 3 – Samples of Student's Wo

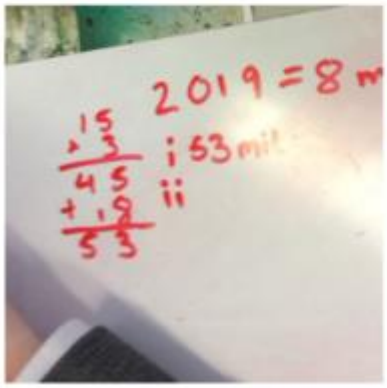

How many tonnes of plastic will be there in 15 years?
A) 120 million tonnes
B) 360 million tonnes
C) 114791256 million tonnes
D) None of the above
E) 1434897 million tonnes

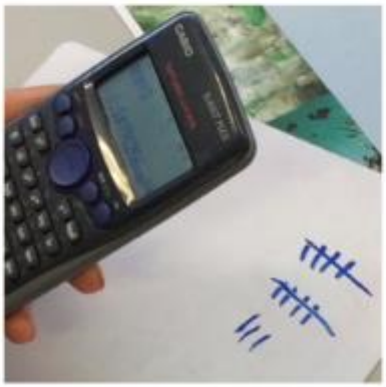

360 million tonnes

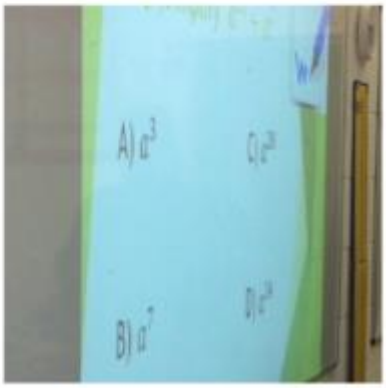

15
x8
120 tons
15 ago

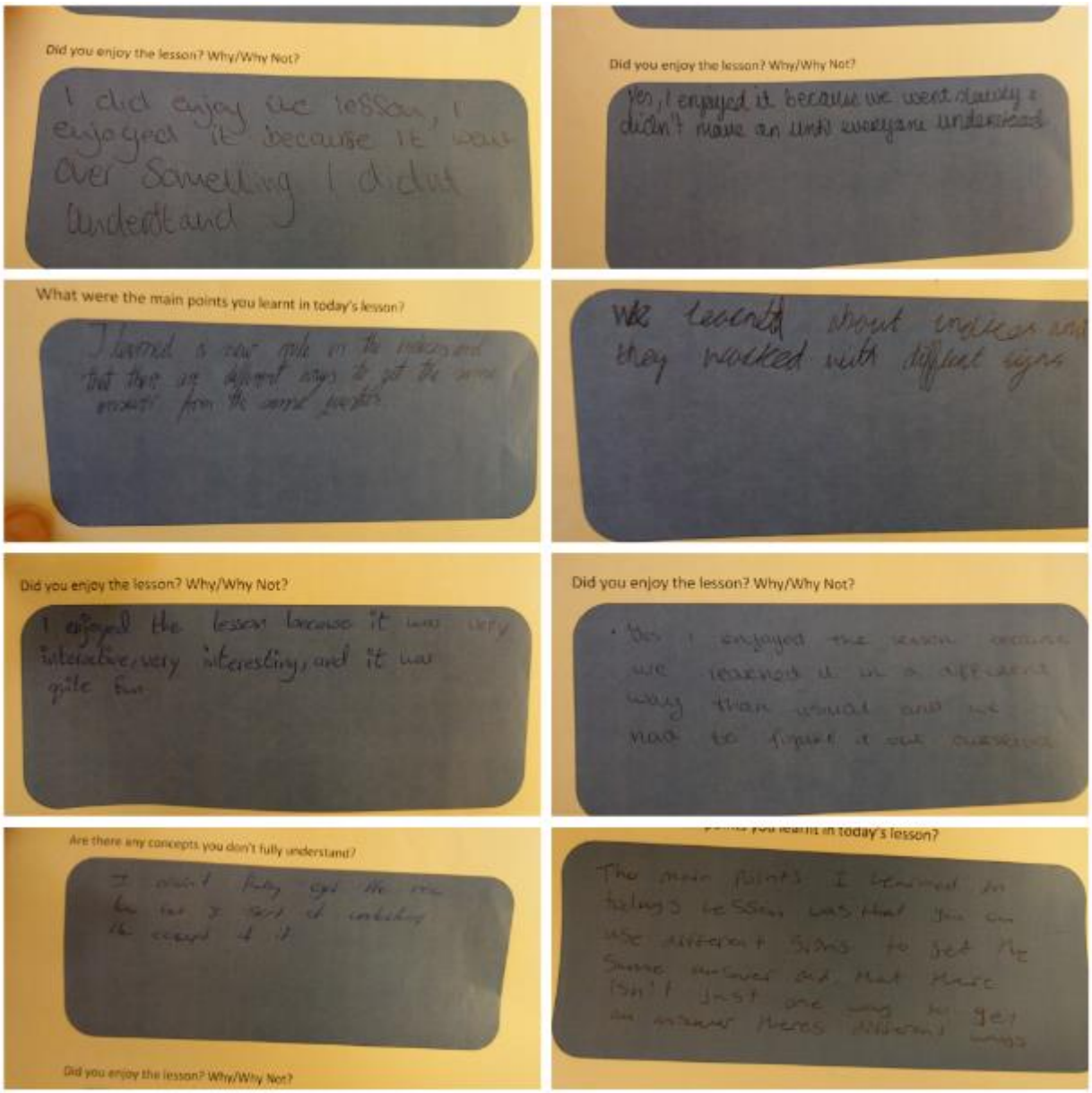

B
A




2019 = 8 m
15
x3
45
+10
55




A) a³
B) a⁷


Did you enjoy the lesson? Why/Why Not?
I did enjoy the lesson, I enjoyed it because it was over something I didnt understand
Did you enjoy the lesson? Why/Why Not?
Yes, I enjoyed it because we went slowly + didn't move on until everyone understood
What were the main points you learnt in today's lesson?
We learnt about angles and they worked with diffent signs
Did you enjoy the lesson? Why/Why Not?
I enjoyed the lesson because it was very interactive, very interesting, and it was quite fun
Did you enjoy the lesson? Why/Why Not?
Are there any concepts you don't fully understand?
Did you enjoy the lesson? Why/Why Not?